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Thermodynamic design of Stirling engine using multi-objective particle swarm optimization algorithm

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ABSTRACT

In the recent years, the interest in Stirling engine has remarkably increased due to its ability to use any heat source from outside including solar energy, fossil fuels and biomass. A large number of studies have been done on Stirling cycle analysis. In the present study, a mathematical model based on thermodynamic analysis of Stirling engine considering regenerative losses and internal irreversibilities has been developed. Power output, thermal efficiency and the cycle irreversibility parameter of Stirling engine are optimized simultaneously using Particle Swarm Optimization (PSO) algorithm, which is more effective than traditional genetic algorithms. In this optimization problem, some important parameters of Stirling engine are considered as decision variables, such as temperatures of the working fluid both in the high temperature isothermal process and in the low temperature isothermal process, dead volume ratios of each heat exchanger, volumes of each working spaces, effectiveness of the regenerator, and the system charge pressure. The Pareto optimal frontier is obtained and the final design solution has been selected by Linear Programming Technique for Multidimensional Analysis of Preference (LINMAP). Results show that the proposed multi-objective optimization approach can significantly outperform traditional single objective approaches.

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1. Introduction

Due to the dramatic consumption of fossil fuels, remarkable attention is drawn to the renewable energy and energy-efficient conversion systems. The world needs a clean energy revolution in order to break dependence on fossil fuels. Such a revolution would enhance globe energy security, promote enduring economic growth and tackle environmental challenges [1]. In this regard, the Stirling engine is one of the most promising sustainable energy technologies in recent years [2–4].

The Stirling engine is a simple type of external combustion engine which operates over a closed, regenerative thermodynamic cycle with the ability to use a wide variety of energy sources such as solar energy, geothermal and industrial waste heat. It plays an important role on environmental protection and also has significance for moderating the pressure on fossil fuels supplies in the world. With ideal regeneration and isothermal processes, the Stirling engine can theoretically convert heat into mechanical work at Carnot efficiency. The thermal limit for the operation of Stirling engine depends on the performance of heat resistant materials used for its construction. In most instances, Stirling engines operate with a heat source and heat sink temperature of 923 and 338 K, respectively [5]. The engine thermal efficiency varies from about 30% to 40% resulting from a typical temperature in range of 923–1073 K, and normal operating speed range from 2000 to 4000 rpm [6–10].

The invention of the Stirling engine preceded modern thermodynamic theory to a degree exemplified by the fact that it was not until fifty years after its invention that a reasonable analysis of the cycle was published. This analysis was made by Gustav Schmidt in 1871 [11] and has been reported in a more comprehensive form in the book of Israel Urieli [12]. The analysis, however, is limited in that it assumes isothermal working spaces and ideal heat exchangers, and thus it calculates the efficiency as being only temperature-dependent (i.e., Carnot efficiency). In 1960, Finkelstein presented an ideal analysis, in which he assumed adiabatic working spaces but maintained the heat exchangers as ideal [13]. The effects of heat transfer and imperfect regeneration on the performance of the irreversible Stirling engine was investigated by Wu et al. [14]. As the efforts to reduce certain loss mechanisms can tend to increase the detrimental impact of others, complementary approaches are needed. It is noted that a practical Stirling cycle always departs significantly from the ideal cycle, mainly

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Nomenclature

C_V	specific heat at constant volume (J/kg K)	T_3	working fluid temperature in the hot space (K)
e	regenerator effectiveness	$T_{3'}$	working fluid temperature at the regenerator outlet (K)
E_S	Stirling engine thermal efficiency	T_1	working fluid temperature in the cold space (K)
K	factor defined by Eq. (9)	$T_{1'}$	working fluid temperature at the regenerator inlet (K)
k	specific heat ratio	T_R	effective working fluid temperature in regenerator dead space (K)
k_{DH}	hot space dead volume ratio	V_{DH}	hot space dead volume (m^3)
k_{DR}	regenerator dead volume ratio	V_{DR}	regenerator dead volume (m^3)
k_{DC}	cold space dead volume ratio	V_{DC}	cold space dead volume (m^3)
m	total working fluid mass contained in the engine (kg)	V_D	total dead volume (m^3)
p	charge pressure (N/m^2)	V_E	expansion volume (m^3)
Q_{in}	total heat added from an external heat source (J)	V_C	compression volume (m^3)
Q_{out}	total heat rejected to an external heat sink (J)	W_{net}	engine net work (J)
R	gas constant (J/kg K)		
R_S	irreversibility parameter		

due to the performance of the heat exchanger and the regenerator. A detailed analysis with practical values and examples of the engine were given by Organ [15]. The similarity and scaling approach developed by Organ provides a simpler way of creating the preliminary design for the prototype engine. In 2000, Kaushik and Kumar presented an investigation of a finite-time thermodynamic analysis of a Stirling engine in which the power output and the corresponding thermal efficiency were maximized [16]. Petrescu and Costea [17] conducted a technique for calculating the efficiency and power of Stirling engines based on the First law of thermodynamics for processes with finite speed [17,18]. In 2008, Timoumi and Tlili developed a second-order Stirling model which includes thermal losses and is applied for the optimization of GPU-3 Stirling engines [19]. Leonardo Scollo [20] presented an alpha-type Stirling engine by using a similar design method. A numerical model for beta-type Stirling engine with rhombic drive mechanism was developed by Cheng et al. [21]. The energy equations of the control volumes in the working chambers and heat exchangers were derived and solved by taking into account the non-isothermal effects, the thermal resistance of the heater head and the effectiveness of the regenerative channel [21]. Formosa et al. [22] developed an analytical model taken into account the heat losses and irreversibilities on Stirling engines which could be used in a dynamical analysis for preliminary design [22]. In 2011, a semi-analytical dynamic model of free piston Stirling engine was developed by Formosa [23]. The thermodynamic model is used to define the thermal variables which are used in dynamic model which evaluates the kinematic results. Recently, Aksoy and Cinar [24] conducted a theoretical investigation on dynamic and thermodynamic analysis of a beta-type Stirling engine with rhombic drive mechanism by implementing nodal analysis method [24]. Solmaz and Karabulut [25] presented a novel configuration of beta-type Stirling engine which was driven by a lever mechanism. The performance of the novel engine was compared with rhombic drive engine via nodal analysis [25]. Abdollahpour et al. [26] developed a thermodynamic model and an optical model of a parabolish dish collector for Stirling engine. The collector has the ability to be coupled with different type of Stirling engine and used in various fields like air conditioning and solar water heater in the following of generate electricity from Stirling engine [26].

Although a large number of studies have been done on the optimization of Stirling cycle and Stirling engine, most of them are merely focused on single impact factor. Previous work has been done on dynamic and thermodynamic analysis of Stirling engine by the authors [27,28]. In this paper, we have studied three objective functions simultaneously: engine power output, thermal efficiency of Stirling engine, and the cycle irreversibility parameter which has previously been proved to be significant in predicting the performance of Stirling engine in the thermodynamic cycle [29]. Meanwhile, ten decision variables including temperatures of heat source and heat sink, volumes of expansion space and compression space, total dead volume, volume ratios of three heat exchangers, effectiveness of regenerator, and mean effective pressure are considered during the multi-objective optimization process.

Problems with multiple objectives are present in a great variety of engineering optimization problems. It is quite common that in these problems there are several conflicting objectives to be optimized and it is difficult to identify what the best solution is. Despite the considerable diversity of techniques in Operation Research developed to tackle these problems, their intrinsic complexity calls for alternative approaches. Recently, Ahmadi et al. [30–35] conducted a series of works on multi-objective optimization method for designing a power Stirling engine by implementing the second-version Non-dominated Sorting Genetic Algorithm (NSGA-II) which is known as a tributary of evolutionary algorithm. In this study, a superior technique that has been adopted for dealing with multi-objective optimization problems is Particle Swarm Optimization (PSO) [36], which is a relatively heuristic inspired by the choreography of a bird flock. This method was applied in the area of designing Stirling engine for the first time. The study developed by Coello [37] demonstrated that PSO is highly competitive and can be considered a viable alternative to solve multi-objective optimization problems. In addition, the exceptionally low computational times required by PSO approach make it a very promising approach to problems (e.g., engineering optimization) in which the computational cost is a vital issue.

2. Stirling system description

The ideal Stirling cycle satisfies the Carnot requirement of reversibility and can be described with reference to Fig. 1. One of the mechanical configurations for realizing the Stirling cycle is shown, being known as the Alpha arrangement. The regenerator generally comprises a matrix of fine wires, porous metal or sometimes simply the metal wall surface enclosing an annular gap.

The position of the pistons are shown at the four extreme state points of the cycle as seen in the pressure-volume and temperature-entropy diagrams. Process 1–2 is the isothermal compression process during which the heat is removed from the engine at the cold sink temperature. Similarly, process 3–4 is the isothermal expansion process during which heat is added to the engine at the hot source temperature. Process 2–3 and 4–1 are the constant

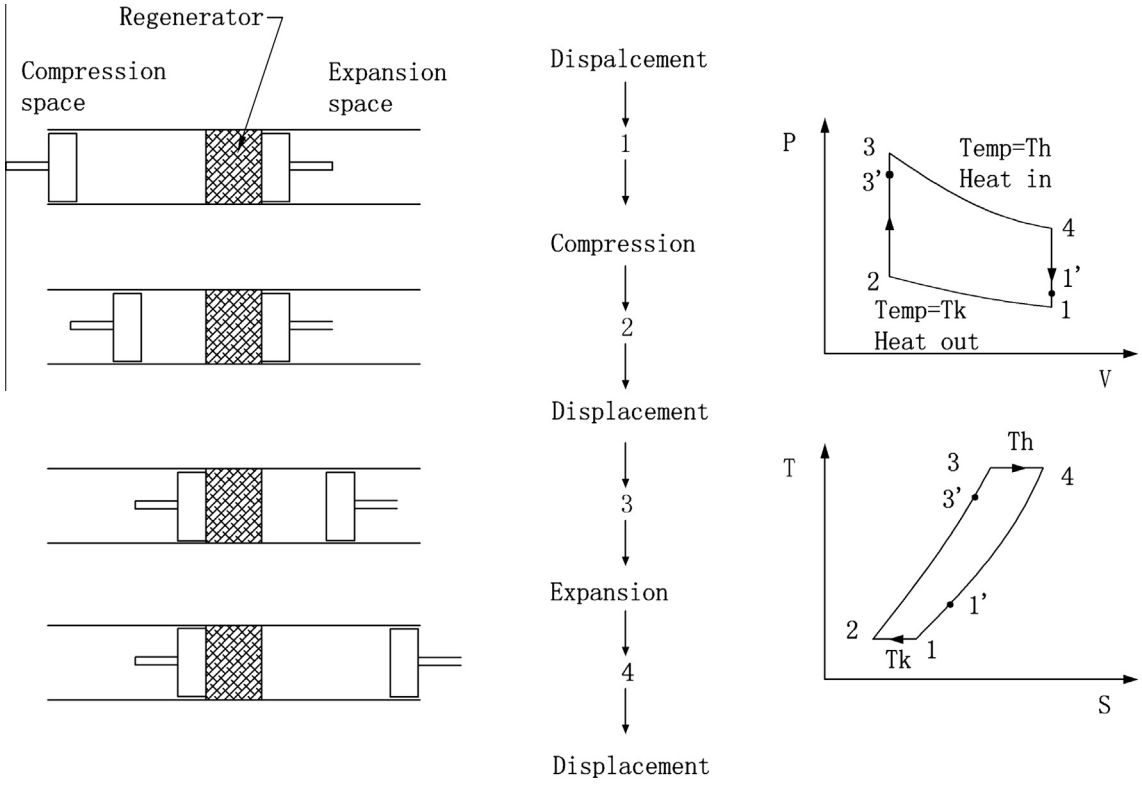

Fig. 1. Illustration of the Stirling cycle system.

volume displacement processes in which the working gas is passed through the regenerator. During the process 4–1 the working gas gives its heat up to the regenerator matrix, and the heat will be recovered from the regenerator matrix to the gas subsequently during the process 2–3. Thus, even though quite considerable quantities of heat are transferred during the displacement process, they are seen to be externally adiabatic, fulfilling the Carnot requirements for maximum attainable efficiency.

3. Analysis of Stirling cycle with internal irreversibility

3.1. Stirling engine dead volumes

Total dead volume includes the heater, the regenerator and the cooler dead volumes and is expressed as:

$$V_D = V_{DH} + V_{DR} + V_{DC} = (k_{DH} + k_{DR} + k_{DC})V_D \tag{1}$$

where k_{DH}, k_{DR} and k_{DC} are the heater dead volume ratio, the regenerator dead volume ratio and the cooler dead volume ratio, respectively.

3.2. Imperfect regenerator

For an imperfect regenerator, the working fluid temperature at the outlet and inlet of the regenerator are $T_{3'}$ and $T_{1'}$, respectively. The regenerator effectiveness, e, is defined as follows [38]:

$$e = \frac{T_{3'} - T_1}{T_3 - T_1} \tag{2}$$

The regenerator effectiveness e thus varies from 0 for no regeneration to 1 for an ideal regenerator. The temperature of the working fluid at the regenerator outlet can be expressed in terms of the regenerator effectiveness as:

$$T_{3'} = T_1 + e(T_3 - T_1) \tag{3}$$

In a similar way, the working fluid temperature at the regenerator inlet can be expressed as:

$$T_{1'} = T_3 + e(T_1 - T_3) \tag{4}$$

For a real Stirling cycle, the accurate description of working fluid temperature for the regenerator is important. The effective temperature of the working fluid within the regenerator dead volume can be determined using the arithmetic mean [38]:

$$T_R = \frac{T_{3'} + T_{1'}}{2} \tag{5}$$

Substituting Eqs. (3) and (4) into Eq. (5) gives:

$$T_R = \frac{T_3 + T_1}{2} \tag{6}$$

3.3. State equation

Assume that the expansion and compression volumes are, V_E and V_C respectively. Using the ideal gas law given by:

$$pV = mRT \tag{7}$$

the equation of state for the isothermal compression process 1–2 with dead volumes V_{DH}, V_{DR} and V_{DC} can be expressed as follows:

$$p = \frac{mR}{\left(\frac{V_E}{T_3} + \frac{V_{DH}}{T_3} + \frac{V_{DR}}{T_R} + \frac{V_{DC}}{T_1} + \frac{V_C}{T_1}\right)} = \frac{mR}{\left(\frac{V_E}{T_3} + K + \frac{V_C}{T_1}\right)} \tag{8}$$

where

$$K = \frac{V_{DH}}{T_3} + \frac{V_{DR}}{T_R} + \frac{V_{DC}}{T_1} \tag{9}$$

It should be noted that, for a given hot-side and cold-side working fluid temperature, the factor K is a function of the dead volumes.

3.4. Thermodynamic processes of Stirling cycle

For the isothermal compression process 1–2, the heat rejected is given by:

$$Q_{1-2} = W_{1-2} = \int_{V_E+V_C}^{V_E} pdV_C = mRT_1 \ln\left[\frac{(V_E+KT_1)}{(V_E+V_C+KT_1)}\right] \quad (10)$$

It can be seen that the compression work depends only on the factor K which is a function of the dead volumes.

For the isochoric heating process 2–3, the heat added is given by:

$$Q_{2-3} = mC_V(T_3 - T_2) = mC_V(T_3 - T_1) \quad (11)$$

where C_V is the specific heat at constant volume and is assumed to be constant. Without regeneration, this amount of heat is added from an external heat source, while for ideal regeneration this amount of heat is released from an ideal regenerator.

The regeneration heat released from an imperfect regenerator during the process 2–3' is:

$$Q_{2-3'} = mC_V(T_{3'} - T_2) = emC_V(T_3 - T_1) \quad (12)$$

The heat added from the external heat source during process 3'-3 is:

$$Q_{3'-3} = mC_V(T_3 - T_{3'}) = (1-e)mC_V(T_3 - T_1) \quad (13)$$

It is clear that the heat input to the isochoric process depends only on the regenerator effectiveness.

In the isothermal expansion process 3–4, the heat added to the cycle is given by:

$$Q_{3-4} = W_{3-4} = m\int_{V_E}^{V_E+V_C} pdV_E = mRT_3 \ln\left[\frac{(V_E+V_C+KT_3)}{(V_E+KT_3)}\right] \quad (14)$$

It is obviously that expansion work depends on the dead volumes.

For the isochoric cooling process 4–1, the heat rejected is:

$$Q_{4-1} = mC_V(T_1 - T_4) = -mC_V(T_3 - T_1) \quad (15)$$

Without regeneration, this amount of heat is rejected to an external heat sink, while for ideal regeneration this amount of heat is rejected to the ideal regenerator.

For an imperfect regeneration, the heat rejected to the regenerator is given by:

$$Q_{4-1'} = mC_V(T_{1'} - T_4) = -emC_V(T_3 - T_1) \quad (16)$$

The heat rejected to an external heat sink during process 1'-1 is given by:

$$Q_{1'-1} = mC_V(T_1 - T_{1'}) = -(1-e)mC_V(T_3 - T_1) \quad (17)$$

It is evidenced that the heat transfer during the cooling process depends only on the regenerator effectiveness.

Consequently, for an imperfect regeneration, the total heat added and heat rejected in Stirling cycle can be expressed as follows:

$$Q_{in} = Q_{3-3'} + Q_{3-4} = mC_V\left\{(1-e)(T_3 - T_1) + (k-1)T_3 \ln\left[\frac{(V_E+V_C+KT_3)}{(V_E+KT_3)}\right]\right\} \quad (18)$$

$$Q_{out} = Q_{1-1'} + Q_{1-2} = -mC_V\left\{(1-e)(T_3 - T_1) + (k-1)T_1 \ln\left[\frac{(V_E+V_C+KT_1)}{(V_E+KT_1)}\right]\right\} \quad (19)$$

The engine net work can be determined from:

$$W_{net} = \sum Q = Q_{in} - Q_{out} = p\left(\frac{V_E}{T_3} + K + \frac{V_C}{T_1}\right)\left\{T_3 \ln\left[\frac{(V_E+V_C+KT_3)}{(V_E+KT_3)}\right] - T_1 \ln\left[\frac{(V_E+V_C+KT_1)}{(V_E+KT_1)}\right]\right\} \quad (20)$$

The engine thermal efficiency can be determined from:

$$E_S = \frac{W_{net}}{Q_{in}} = \frac{\left\{T_3 \ln\left[\frac{(V_E+V_C+KT_3)}{(V_E+KT_3)}\right] - T_1 \ln\left[\frac{(V_E+V_C+KT_1)}{(V_E+KT_1)}\right]\right\}}{\left\{T_3 \ln\left[\frac{(V_E+V_C+KT_3)}{(V_E+KT_3)}\right] + \frac{(1-e)(T_3-T_1)}{k-1}\right\}} \quad (21)$$

In order to describe quantitatively the effect of the internal dissipations of the working fluid on the performance of Stirling engine, the cycle irreversibility parameter, R_S, is defined as:

$$R_S = \ln\left(\frac{V_D+V_P+KT_3}{V_D+KT_3}\right) \Big/ \ln\left(\frac{V_D+V_P+KT_1}{V_D+KT_1}\right) = \frac{s_4 - s_3}{s_1 - s_2} \quad (22)$$

It shows that the internal irreversibilities in a Stirling engine can be characterized by a single parameter representing the ratio of two entropy differences. This parameter has not been widely adopted in designing Stiring engine. In this study, R_S is considered as one of the objectives to be maximized. It has been proved in [29] that Stirling engine with higher the cycle irreversibility can deliver more power and has higher efficiency.

4. Multi-objective optimization via particle swarm optimization algorithm

4.1. Multi-objective particle swarm optimization

The optimization of Stirling engine is a typical multi-objective optimization problem. Let $S \subset \Re^n$ be an n-dimensional search space, and $f_i(\vec{x}), i = 1,\ldots,k$ be k objective functions which are defined over S. Also let f be a vector function defined as, $f(\vec{x}) = [f_1(\vec{x}), f_2(\vec{x}), \ldots, f_k(\vec{x})]$, and $g_i(\vec{x}) \leqslant 0, i = 1,\ldots,m$ be m inequality constraints. Then the goal of multi-objective optimization is to find a solution, $\vec{X}^* = (X_1^*, X_2^*, \ldots, X_n^*)$, that minimizes $f(\vec{x})$.

To solve this problem, an advanced technology in optimization is adopted, which is Multi-objective Particle Swarm Optimization (MOPSO) [37]. MOPSO is a general optimization algorithm, but it has never been applied in the research area of Stirling engine. In this study, MOPSO is proved to work extremely well in task of designing Stirling engine.

In order to introduce MOPSO, a review of Particle Swarm Optimization (PSO) algorithm is given. As a classic optimization technique, PSO was first proposed by Kennedy and Eberhart in 1995 [36]. It is a computational method that optimizes a problem by iteratively trying to improve a candidate solution with regard to a given measure of quality, e.g., an objective function in our case. This method is similar in some aspects to other evolutionary algorithms with the exception of that particles move rather than evolve through the search space. If enough particles are given, PSO guarantees to get the global optimum solution of the objective function. In practice, however, fewer particles are used to find a balance between accuracy and efficiency, which usually leads to suboptimum solution.

As compared to Genetic algorithms (GA), there are two main differences between PSO and GA [39]. Firstly, genetic algorithms rely on three mechanisms in their processing: parent representation, selection of individuals and the fine tuning of their parameters. On the contrary, PSO only relies on two mechanisms because it does not adopt an explicit selection operator. However, the absence of a selection mechanism in PSO is compensated by the use of leaders to guide the search. A second distinction has to do with the way in which the individuals are manipulated. PSO uses an operator that sets the velocity of a particle to a particular direction. This can be seen as a directional mutation operator in

which the direction is defined by both the particle's personal best and the global best of the swarm. If the direction of the personal best is similar to the direction of the global best, the angle of potential directions will be small, whereas a larger angle will provide a larger range of exploration. In contrast, GA uses a mutation operator that can set an individual in any direction, although the relative probabilities for each direction may be different.

In multi-objective optimization, the objectives to be optimized are normally in conflict to each other, which means that there is no single solution for these problems. Instead, MOPSO aims to find good tradeoff solutions that represent the best possible compromises among the objectives based on the concept of Pareto optimality. A point $\vec{X}^* \subset \Omega$ is Pareto optimality if for every $\vec{X} \subset \Omega$ and $I = \{1, 2, \ldots, k\}$ either

$$\forall_{i \subset I}(f_i(\vec{X}) = f_i(\vec{X}^*)) \tag{23}$$

or, there is at least one $i \subset I$ such that

$$f_i(\vec{X}) > f_i(\vec{X}^*) \tag{24}$$

It is defined that X^* is Pareto optimal if there exists no feasible vector X which would decrease some criterion without causing a simultaneous increase in at least one other criterion. In MOPSO, all objective functions are evaluated for each particle. Pareto optimality can guide the particles to produce nondominated best positions (often called *leaders*). Since there can be many nondominated solutions in the neighborhood of particle, the determination of leaders is not straightforward, but only one is usually selected to participate in the velocity update in PSO. The whole scheme of MOPSO can be outline in Fig. 2, in which, the external archive is used for storing the nondominated solutions discovered during search [40–42]. An external archive has bounded size, thereby making unavoidable the imposition of the rules regarding the replacement of existing solutions with new ones.

4.2. Objective functions, decision variables and constraints

In the present study, three key objective functions for optimization are the output power (W_{net}), the Stirling engine thermal efficiency (E_S) and the cycle irreversibility parameter (R_S), denoted by Eqs. (20)–(22), respectively. There are ten decision variables considered as follows:

T_1: Working fluid temperature in the cold space
T_3: Working fluid temperature in the hot space
V_D: Total dead volume
V_E: Expansion volume
V_C: Compression volume
e: Effectiveness of regenerator
k_{DH}: Hot space dead volume ratio
k_{DR}: Regenerator dead volume ratio
k_{DC}: Cold space dead volume ratio
p: System charge pressure.

The objective functions have been solved by implementing the MOPSO algorithm described above with respect to the following constrains:

$$288 \leqslant T_1 \leqslant 320 \text{ K} \tag{25}$$

$$800 \leqslant T_3 \leqslant 1300 \text{ K} \tag{26}$$

$$50 \leqslant V_E \leqslant 150 \text{ cm}^2 \tag{27}$$

$$50 \leqslant V_C \leqslant 150 \text{ cm}^2 \tag{28}$$

$$75 \leqslant V_D \leqslant 180 \text{ cm}^2 \tag{29}$$

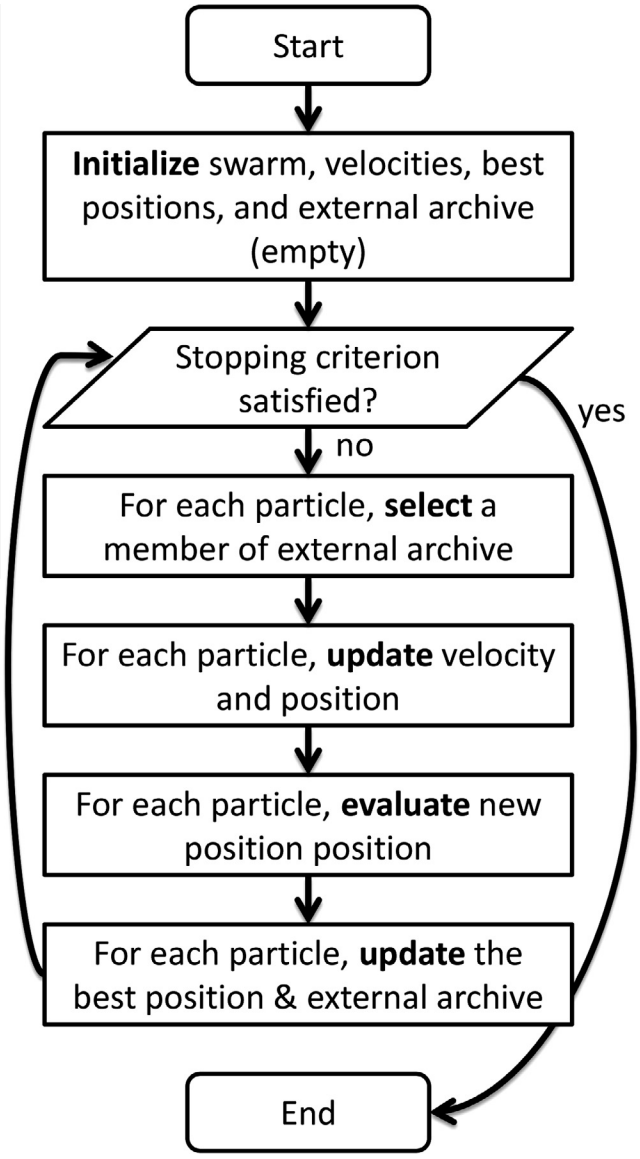

Fig. 2. The flow chart of MOPSO.

$$0.4 \leqslant e \leqslant 0.9 \tag{30}$$

$$0.1 \leqslant k_{DH} \leqslant 0.2 \tag{31}$$

$$0.2 \leqslant k_{DR} \leqslant 0.4 \tag{32}$$

$$0.4 \leqslant k_{DC} \leqslant 0.7 \tag{33}$$

$$k_{DH} + k_{DR} + k_{DC} = 1 \tag{34}$$

$$1 \leqslant p \leqslant 5 \text{ Mpa} \tag{35}$$

In this study, a MOPSO based Matlab program is developed to find the optimum design parameters of the Stirling engine. The population size of MOPSO algorithm options is set to be 250.

5. Decision-making methods in multi-objective optimization

In multi-objective optimization, a decision-making process for selection of the best optimal solution from all the feasible alternatives is required. There exist a number of several methods for deci-

Table 1
Specifications of the Stirling engine.

Work gas	Nom. molar mass	Gas constant	Specific heat		Specific heat ratio
	M (kg/kmol)	R (kJ/kg K)	c_p	c_v	$c_p/c_v = \gamma$
He	4	2.08	5.19	3.11	1.67

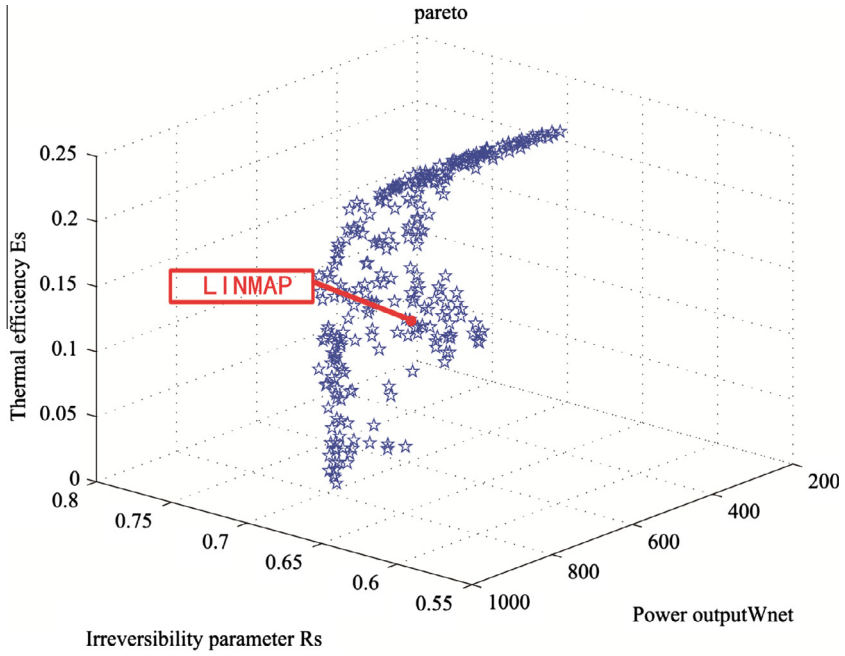

Fig. 3. Pareto optimal frontier in objectives' space.

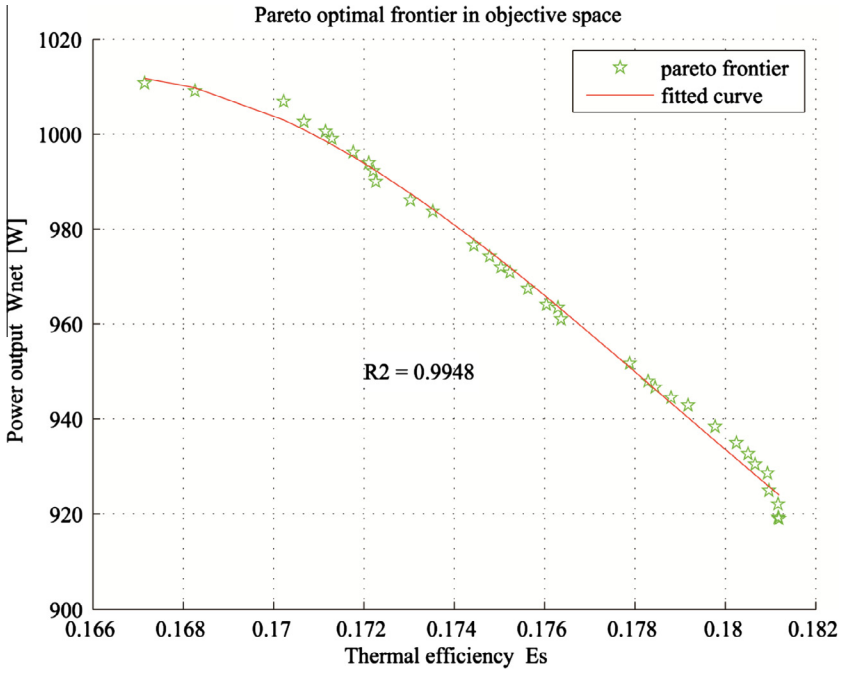

Fig. 4. Pareto optimal frontier in objectives' space ($W_{net} - E_S$).

sion-making process in Decision problems. And these methods can be implemented in decision-making for selection of a final optimal solution from the Pareto frontier which is obtained by MOPSO. Linear Programming Technique for Multidimensional Analysis of Preference (LINMAP), which is one of the most recognized and popular type of decision-making processes, is based on pair-wise solutions' comparisons obtained from decision-making processes and yields the best compromise solution which has the shortest

distance to the positive ideal solution [43,44]. Recently, Ahmadi et al. [31,32] observed that the LINMAP decision-making method could generate a final optimal solution with the least deviation index from the positive ideal solution. Hence, in the present study, LINMAP is implemented and the final optimal solution has been decided based on this method.

6. Results and discussion

The engine power output, thermal efficiency of Stirling engine and irreversibility parameter are maximized simultaneously by implementing multi-objective optimization method based on particle swarm optimization algorithm.

In this study, the optimization process is conducted with three objective functions which are expressed by Eqs. (20)–(22) and ten design parameters that are restrained by Eqs. (25)–(35). The decision variables (design parameters) of optimization are temperature of working fluid in the hot space, temperature of working fluid in the cold space, total dead volume, expansion volume, compression volume, dead volume ratio of the hot space, dead volume ratio of the cold space, dead volume ratio of the regenerator, effectiveness of the regenerator, and the system charge pressure. In this case,

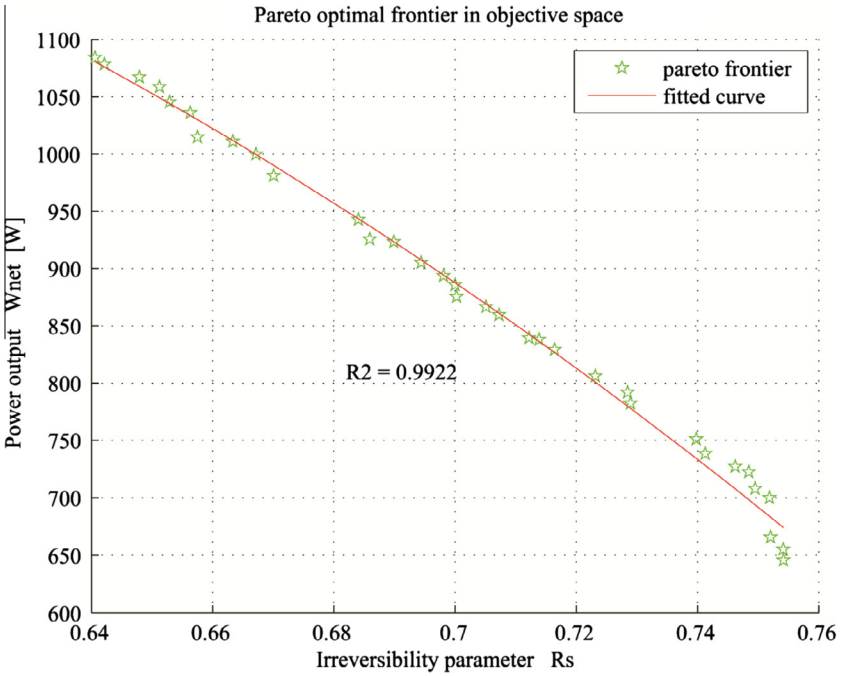

Fig. 5. Pareto optimal frontier in objectives' space ($W_{net} - R_S$).

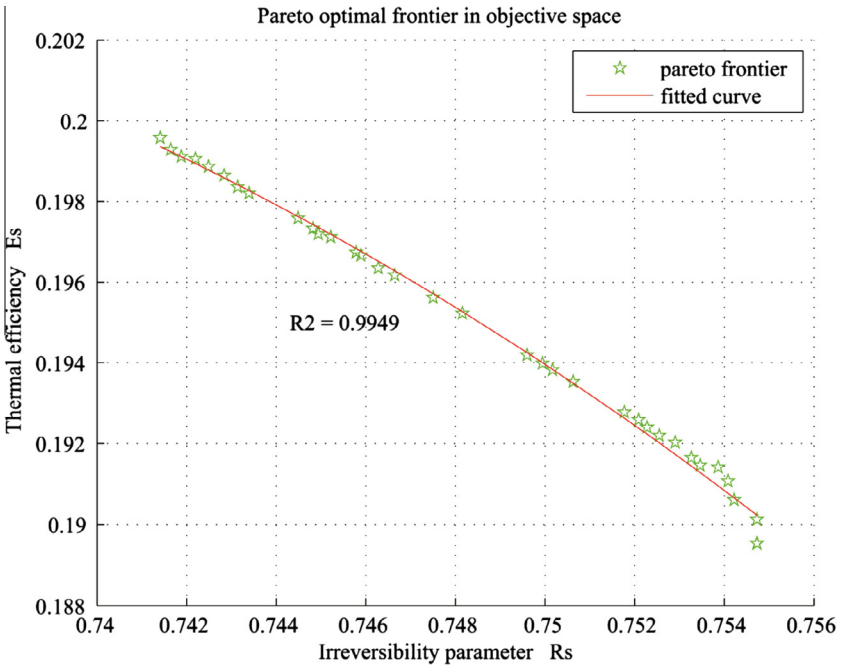

Fig. 6. Pareto optimal frontier in objectives' space ($E_S - R_S$).

Table 2
Final optimal solution obtained by LINMAP decision-making method.

Methods	Decision variables										Objective functions		
	T_1 (K)	T_3 (K)	V_D (cm^3)	V_E (cm^3)	V_C (cm^3)	e	k_{DH}	k_{DR}	k_{DC}	P (Mpa)	*Power* (W)	E_S	R_S
LINMAP	297.9	914.1	143.3	98.1	75.9	0.89	0.38	0.41	0.21	4.61	732.3	0.18	0.67
Results [28]	314	923	136.2	96.8	93.1	0.88	0.4	0.5	0.1	4.1	633.6	0.15	–

helium is chosen as the working fluid. In order to have consistency with previous works [28], specifications of the Stirling engine are summarized in Table 1.

The Pareto optimal frontier in the proposed objectives' space obtained by multi-objective particle swarm optimization is illustrated in Fig. 3. The final solution selected by LINMAP decision-maker is also indicated in Fig. 3. Moreover, a surface is fitted to the points obtained by multi-objective optimization as following:

$$E_S = 6.25\times10^4 - 1.09\times10^5 R_S - 2.69\times10^5 W_{net} + 2.43 \times 10^5 R_S^2 + 2.48\times10^5 R_S W_{net} + 4.03\times10^5 W_{net}^2 - 2.77 \times 10^5 R_S^3 - 2.39\times10^5 R_S^2 W_{net} - 1.38\times10^5 R_S W_{net}^2 - 2.05\times10^5 W_{net}^3 \quad (36)$$

which can be applied to gain the irreversibility parameter.

In Fig. 4 Pareto optimal frontier for objective functions of power output and thermal efficiency of the Stirling engine is depicted. As can be seen in Fig. 4, the power output of the Stirling engine decreases when the thermal efficiency increases. In addition, the fitted curve equation of Pareto optimal frontier is obtained as following:

$$W_{net} = 1.77\times10^7 E_S^3 - 9.48\times10^6 E_S^2 + 1.69\times10^6 E_S - 98479.11 \quad (37)$$

Pareto optimal frontier for power output and irreversibility parameter is presented in Fig. 5. It can be seen that engine power output decreases with increasing the irreversibility parameter of the Stirling cycle. To assist the optimal design of the Stirling engine, the fitted curve, which is valid in the range of $0.64 < R_S < 0.75$, is derived as following:

$$W_{net} = -6068.98 R_S^2 + 4886.83 R_S + 440.56 \quad (38)$$

Fig. 6 illustrates the Pareto optimal frontier for thermal efficiency and irreversibility parameter of the Stirling engine. It is clear that thermal efficiency increases to 0.198 when irreversibility parameter decreases to 0.742. The fitted curve equation of Pareto optimal frontier is expressed as following:

$$E_S = -12.03 R_S^2 + 17.31 R_S - 6.02 \quad (39)$$

The final optimal solution for design parameters and objective functions using LINMAP decision-making method is presented in Table 2 compared with the experimental data [28] in which the cycle irreversibility parameter R_S is not considered. The improved design outperforms the previous work [28] by more than 15% and 20% on engine power output and thermal efficiency, respectively. It is obviously that the design proposal obtained from the proposed MOPSO algorithm leads to better performance of Stirling engine than that of the traditional design.

7. Conclusions

According to the previous study on the Stirling cycle and Stirling regenerator, the thermodynamic analysis of the Stirling engine proposed in this paper shows more accurate results compared with the others due to the consideration of the cycle irreversibility parameter. The engine power output, thermal efficiency and the cycle irreversibility parameter are optimized simultaneously for the multi-objective optimization while temperature of working fluid in the hot space (T_1), temperature of working fluid in the cold space (T_3), total dead volume (V_D), expansion volume (V_E), compression volume (V_C), dead volume ratio of the hot space (k_{DH}), dead volume ratio of the cold space (k_{DC}), dead volume ratio of the regenerator (k_{DR}), effectiveness of the regenerator (e), and the system charge pressure (p) are considered as design parameters. Multi-objective optimization is conducted by implementing particle swarm optimization algorithm and the Pareto optimal frontier in objectives' space is obtained. Moreover, the final solution is selected from Pareto optimal by LINMAP decision-maker. The indicated power output is 732.3 W. The thermal efficiency and the irreversibility parameter of the Stirling engine is 0.18 and 0.67, respectively. The simulation results show the correctness of our physical model and mathematical model, which suggests that the proposed multi-objective multi-variable optimization method is capable in designing Stirling engine. The performance of Stirling engine designed by MOPSO method is 15% and 20% better than previous work based on traditional single-objective approach on power output and thermal efficiency, respectively.

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